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AXIS OF THE ZODIACAL LIGHT AND THE CONSTANTS OF ITS PLANE OF SYMMETRY

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AXIS OF THE ZODIACAL LIGHT AND THE CONSTANTS OF ITS PLANE OF SYMMETRY

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ABSTRACT: On the basis of numerous observations by himself and others, the author begins by investigating the disturbances which affect the axis of the zodiacal light owing to atmospheric extinction, twilight, atmospheric skylight and the Milky Way. A portion of the disturbing factors can be overcome by corrections. Where this is not possible, the magnitude of the disturbance is measured so that its influence on the data can be estimatated. The best possible position of the light axis, extended over all seasons of the year, can then be used for a new determination of the constants of the plane of symmetry, as previously defined in an earlier paper. The authors previous values of the constants, based primarily on observations by Jones, are generally confirmed and occasionally improved. A detailed description of the contents of the paper is to be found in the last section. /49*

1. Introduction

On an arc of a great circle, which intersects the ecliptic at a right angle, a few points can always be found at which the brightness of the zodiacal light reaches a maximum. The axis of the zodiacal light consists of all the points obtained in this manner at all ecliptic longitudes. It is particularly important to stress this definition, because (in addition to the light axis obtained in this manner) it is possible to have a geometric or topographic axis which acts as the bisectrix of the planes cut out by the outlines of the zodiacal light, regardless of whether these outlines are true isophots or not. The two axes will not be completely coincidental even if we disregard the disturbance of the zodiacal light by extinction, skylight, twilight, and the Milky Way.

In our first paper [1], we properly stressed the great significance of the light axis for the theory of the zodiacal light. At that time, in 1932, we had to content ourselves primarily with the available observations, which did not always correspond to the requirements which one must satisfy and which became clearly

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* Numbers in the margin indicate pagination in the foreign text.

evident to a certain degree only during the course of that work. With respect to the perihelial portion of the zodiacal light (ZL), up to 100° easterly and westerly elongation, our investigations were based particularly on the observations of Jones. Even Jones, however, plotted outlines almost exclusively, and left it to his co-workers to determine the axis. This task was made somewhat easier by the fact that Jones drew the outlines relatively narrow and preferred the narrow form of the ZL [1, p. 38]. This fact, coupled with the general reliability of his drawings, made it possible to use his work as the basis of very extensive research into the shape and position of the plane of symmetry of the inner portion of the ZL. At the same time, however, it was necessary to test these results in a comprehensive series of observations, which would be free of influences as much as possible, and which would yield the points on the axis directly. We are now in a position to describe the data obtained from such a series of observations.

2. Observations

(a) Nature and Scope of the Material.

The position of maximum brightness of the ZL is estimated on the line connecting two stars located on different sides of the light axis and is then transferred to the star chart. In this manner, we ensure that the individual points on the axis are independent of one another; this is not the case if the observer enters a closed axis path in the map.

The available series of observations are listed in Table 1. In addition to our own series, which were obtained on a second expedition in 1933, in Sonneberg, and in Windhuk, five other series were examined, of which one is so comprehensive that it constitutes one of the important supports for this investigation. This series, observed by Wolff in Jerusalem, is especially valuable because it forms the northern counterpart of our Windhuk series. /50

TABLE 1: OBSERVATION SERIES

No.	Location (φ)	Observer	Date	No. of Obs.	Symbol of Series
1	Atlantic Ocean and Bordering Seas (+46° to -35°)	Hoffmeister	1933	610	2FF
2	Atlantic Ocean and Bordering Seas (+46° to -35°)	Richter	1933	189	2FF (R)
3	Sonneberg (+50.4°)	Hoffmeister	1933–40	393	So
4	Jerusalem (-31.8°)	Wolff	1933–36	2028	J
5	Windhuk (-22.6°)	Hoffmeister	1937–38	1234	Wi
6	Indian Ocean (+8° to -21°)	Hoffmeister	1938	8	3FF
7	Tropical and Sub-Tropical Seas (+38° to -5°)	Capt Drent	1925–30	307	D
8	Egypt (+24° to +26°)	Sandner	1928	6	S1
9	Atlantic Ocean, West Africa (+20° to +38°)	Sandner	1938	16	S2

Remarks on the Series

Series 1, 2, 3, 5, and 6 were obtained in strict accord with the method described above.

Series 4. In 1933, Wolff came to us as a member of the "Bund der Sternfreunde" [Amateur Astronomers' Society], seeking information regarding observation, and we recommended that he observe the ZL using my method. With the exception of attempts made during the first month, he conducted his observations strictly according to this rule. His points are relatively close together, but one gets the impression that they are primarily independent of one another. The series is of great importance both in terms of its scope and its quality.

Series 7. Drent made his observations of the ZL during his trips as captain of a Dutch merchant vessel. The series takes the form of descriptions of the location of the axis points relative to stars; it is very complete and very accurate.

Series 8 and 9. These two short series were made by Dr. Sandner during a trip to Egypt and the Tropics. Series 8 lists a few locations of the "peaks" of the principle illumination, relative to the Pleiades; series 9 gives descriptions of the positions of the axis locations relative to various stars.

The middle of the Gegenschein as such is indicated in Series 1, 2, 3, 5, 6, and 7. The observer in Series 4 also gives the weakest portion of the zodiacal band regularly, as well as locations of the axis in the Gegenschein region, but does not mention the Gegenschein.

The equinox of the maps was 1925.0.

The 4791 axis positions in the series mentioned above was supplemented by 143 observations from our first expedition in 1930 (1 ff) [1, p. 105]; this was a portion of the observations that was made south of + 40° latitude. This meant that almost 5000 axis locations were available. Of course these could not be used completely for all of the desired purposes. If they were obtained at an unsatisfactory axis position, they were useful for investigating the observation error; however this applies only to a small portion of the material, obtained primarily at low northern and southern latitudes.

Of the total of 4934 observations, 2388 (48.5%) are ours, 2028 (41.1%) are from Wolff in Jerusalem. Since both series were observed using the same method, the entire material can be considered to be uniform throughout; this is not an insignificant matter in view of the difficulty of these observations.

(b) Treatment of the Observations.

Special grids were used to read the ecliptic longitudes in full degrees and the latitudes of the axis points in tenths of a degree on the maps, which had neither the equatorial grid nor the ecliptic. The solar longitude was determined in full degrees and in tenths of a degree only for the Gegenschein points. This gave the sidereal time and the angle between the ecliptic and the vertical of its horizon points; this angle is simultaneously the minimum zenith distance which any point on the ecliptic has at a given time. For the fixed observation points Windhuk, Jerusalem and Sonneberg, we have calculated tables which were used to determine the desired angle as a function of the sidereal time.

3. The Gegenschein

(a) Annual Path of the Gegenschein.

As we determined earlier [1, p. 62], the path of the Gegenschein in first approximation is a sinusoid with the ecliptic as the axis and the elements $\Omega = 100°$, $i = 1°-2°$. The problem is to determine the path more precisely, using the newly available richer and better observations. /51

TABLE 2. LIST OF THE OBSERVATIONS OF THE GEGENSCHEIN

Dates	Location	No. of Observations	No. of Days
1. 1930: Feb 3-Mar 6	Atlantic Ocean, Caribbean Sea (+10° to +37°)	13	13
2. 1933: Mar 20-May 23	Atlantic Ocean, Gulf of Mexico (-35° to +35°)	104	28
3. 1933: Aug 20,1940 April 8	Sonneberg (+50°)	55	40
4. 1937: May 9,1938 Feb 2)	Southwest Africa (-23°)	66	63
5. 1938: Feb 27-Mar 8	Indian Ocean (-21° to +8)	4	4

The path of the Gegenschein must be determined from observations which are free from marked influences of extinction and other disturbing factors. Since only the ecliptic latitudes and not the longitudes are used, the Gegenschein cannot be observed in the vicinity of the horizon. In any case, it is generally sufficient to select those observations in which the ecliptic run sufficiently steeply through the horizon, without regard for the distance of the Gegenschein from the zenith. We are referring to the fact that the majority of the observations of the Gegenschein were made at a short distance from the zenith, with inclinations of the ecliptic relative to the vertical up to 25° being allowed; however it developed later that the limit should really be selected somewhat more narrowly.

The observations in Groups 1, 2, 4 and 5 in Table 2, to the extent that they fulfill the abovementioned condition, were used in calculating the average values in Table 3.

TABLE 3. NORMAL POSITIONS OF THE BRIGHTEST SPOT IN THE GEGENSCHEIN.

	ecliptic coordinates λ	ecliptic coordinates β	No. n	weight p
1	136°6	+0°02	4	1
2	159.9	+1.85	11	3
3	181.1	+2.21	10	2
4	188.2	+1.34	20	5
5	193.3	+1.12	17	4
6	211.4	+0.29	7	2
7	221.3	+0.85	19	5
8	232.4	+1.00	11	3
9	308.2	−0.23	9	2
10	318.3	+0.02	8	2
11	336.3	−0.34	8	2
12	345.3	−0.20	10	3
13	11.5	−1.06	10	3

TABLE 4. AVERAGE VALUES OF $\lambda - \odot$ FOR THE GEGENSCHEIN.

Observational Series	Date	n	$\lambda -$
Second Expedition	1933	48	178°95
Windhuk	1937-38	42	178°81
Sonneberg	1933-40	38	178°86

A consideration of the figures shows that the axis of the sinusoid is not the ecliptic but rather a parallel circle which lies to the north. The obvious method of balancing three unknowns (the nodes Ω, the apparent inclination i' and the latitude of the axis asymmetry) could not be used because the most likely interpretation which one can give to the observed phenomena requires that the nodes with the ecliptic must be 180° apart and that only the northerly deviation which the Gegenschein reaches is greater than the southerly. The path of the Gegenschein would therefore not be a pure sinusoid. Unfortunately, the presence of the Milky Way makes it impossible to observe the Gegenschein in the vicinity of its nodes; otherwise, we could determine whether the interpretation given below is correct.

We will begin by aligning an average sinusoid with the ecliptic as the axis, using the method of least squares, and obtain the following result:

$$\Omega = 111°5 \pm 20°4 \qquad i' = 1°10 \pm 0°18$$

If we now separate the observations which were made in the portions of the annual path lying to the north of the ecliptic and to the south of the ecliptic, we obtain the following values of i':

Normal Position	$\lambda - \Omega$	i'
1–8	0°–180°	$1.36° \pm 0.21°$
9–13	180°–360°	$0.81° \pm 0.23°$

We have already mentioned [1, p. 68] the fact that the maximum northern latitude of the Gegenschein is greater than the southern one, on the basis of observations by Douglass et al. in Arequipa; however, we were unable to draw any conclusions regarding the actual state of affairs at that time. Since our observations in Southwest Africa yielded the same results, there is no longer any reason to doubt the earlier data. The free-analysis of the 25 normal positions in the second Harvard series which were reported in [1, p. 54] gave the following results:

Normal Position	$\lambda - \Omega$	i'
14–21	0°–180°	$2.48° \pm 0.58°$
1–13, 22–25	180°–360°	$1.93° \pm 0.18°$

In the above, it should be noted that both observation series were made primarily at lower southern latitudes; hence, an extinction effect would always have shifted the path of the Gegenschein toward the south and never toward the north. The following values for the average parallel of the path of the Gegenschein were obtained from the maximum northern and southern latitudes: + 0.15° according to Hoffmeister, + 0.27° according to Harvard (i.e., an average of + 0.2°). /52

Here too the possibilities of an explanation of this phenomenon should be stated. Already in our first study, we came to the conclusion [1, p. 67] that the assumption of an eccentricity of the ring would provide an easy explanation for the observations. The alternative (that the plane of the ring does not pass through the Sun) could scarcely be taken seriously.

It is noteworthy that the values of i' from the Harvard observations are approximately twice as large as those from our own observations. The possibility that the path of the Gegenschein could have changed between 1890-91 and 1930-40 certainly exists, but is not very probable, since there is another possible explanation. We believe that the reason for this difference lies in a variation of the observation method. We, ourselves, have always tried to determine the point of maximum brightness and have entered only this point on the map. The Harvard observers, on the other hand, did not always work in a uniform manner; however, as we see from the reports by A. Searle [2, p. 15], outlines were generally drawn on the charts and the locations of the middle of the Gegenschein were later determined on the basis of these outlines. The midpoint determined in this topographic manner coincides with the point of maximum brightness, however, only if the line to the mass center of a transverse section coincides with the line to the point opposite the Sun (phase angle 0°). If this is not the case, the effect of the phase law (even with a mass distribution symmetric to the principle plane) will generally cause the point of maximum brightness to lie closer to the ecliptic than the topographic midpoint. In one case, the observer at Arequipa differentiated between the two points, as the following remark by Searle [2] shows: "Of the two positions recorded by Mr. Douglass for 1892, March 29, at 17 hr 18 min GMT, the first relate to the center of the observed light, the second to its brightest portion". The corresponding latitudes are + 3.8° and + 0.3°, which agree with the effect which we would expect. The observation took place at the time of the maximum northerly latitude of the Gegenschein, where the effect is strongest. For the same season, we find the following remarks in our notes: "1933, March 20, ϕ = + 35°; the Gegenschein appears to be more sharply delimited in the south and less in extent than in the north". "1933, March 21, ϕ = + 30°; we once again had the impression that the brightest point in the Gegenschein lies south of the geometric mean. The position data are based on the brightest spot". Hence, it is very likely that we can obtain a correct understanding of the situation if we assume that the elements determined on the basis of our observations for the point of maximum brightness are approximately equivalent to those determined from the Harvard observations for the mass center of the corresponding transverse section. Only the latter values are suitable for determining the distance using the method given by us previously [1, p. 58]. We did not attempt to use the Harvard observations to determine the node longitudes, since the accumulations between 0° and 20° as well as 200° and 230° longitude and the sparce occupation of other longitudes are not suitable. The representation shown in [1, p. 62, Fig. 18] shows that the corresponding path, with respect to the node line as well, lies close to the orbit of Jupiter.

(b) Atmospheric Disturbances of the Locations of the Gegenschein.

After the true annual path of the Gegenschein has been obtained, we can compare the points observed under unfavorable geographical conditions with the points calculated on the basis of the path elements, and thereby determine the error of those observations. Limiting this consideration to our own observations, we obtained the error distribution shown in Figures 1 and 2, where the zenith distance of the Gegenschein is presented as the argument with limitation to an hour angle up to 30° to either side of the meridian and a positive deviation means that the corresponding observation satisfies the extinction rule; hence, the observed point, affected by the atmosphere, therefore lies at a smaller zenith distance than the calculated point. In view of the use of individual observations, the scatter is slight (scarcely greater than ±1°). The small number of observations in Figure 2 can be explained by the fact that the majority of observations was made under satisfactory conditions. Figure 3 gives average values from five observations; only the values at 24° and 26° zenith distance were determined from nine and seven different values, respectively, and partially from observations which were used in averaging the annual path (20° ⩽ z' ⩽ 25°). It was noted above that the minimum zenith distance of the ecliptic Z' was set somewhat too high at ⩽ 25°. Therefore these two average values were corrected by + 0.05°, the error which is to be assumed for the latitudes included owing to incomplete compensation for extinction. The average values of Z' of the observations used for the annual path are 12.75° for the second expedition and 10.05° for Windhuk. In the case of Windhuk, however, an effectiveness of the residual extinction mus be assumed; for the expedition, on the other hand, the error must be compensated by shifting the observation point to the north and south of the ecliptic. In view of the small size of the residual error, this could also be approximately true for Arequipa, which is still located rather far to the north of the Southern Tropic. The latitude of +¯0.2° determined above for the apparent axis of symmetry of the path of the Gegenschein is therefore not markedly changed. /53

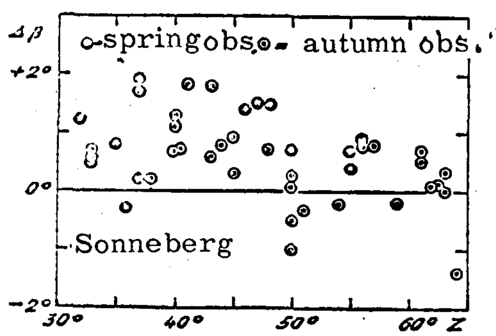


Fig. 1

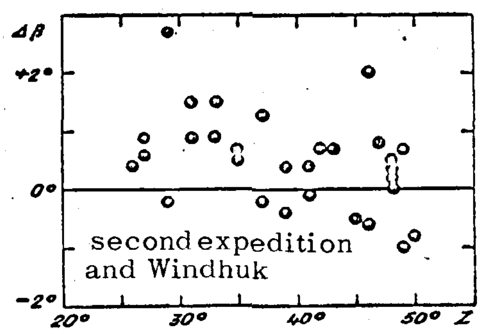


Fig. 2

Errors in the Ecliptic Latitudes of the Gegenschein in Observations Made under Unsatisfactory Geographical Conditions (z > 25°), Positive When the Error Corresponds to the Effect of Extinction.

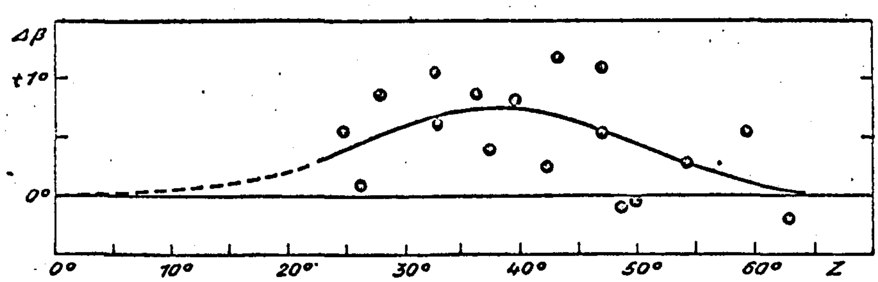


Fig. 3. Average Values and Pattern of Errors Caused by Extinction and Skylight in the Gegenschein.

The most notable fact in Figures 1, 2 and 3 is that the errors reach a maximum (0.75°) at 40° zenith distance and decrease so sharply with higher zenith distances that they become approximately zero at 65°; at still greater zenith distances, even negative values might be obtained, so that the observed center of the Gegenschein then lies closer to the horizon than the true center. The explanation of this phenomenon comes from the fact that the diffuse skylight has an effect in addition to the extinction. Both of these disturbing factors increase toward the horizon, but have opposite effects. To a certain degree, only the powerful influence of the skylight is surprising, which can completely offset the influence of the extinction at approximately 65° zenith distance. Of course, it would be erroneous if the corrections based on Figure 3 were to be used as generally valid: in the first place, the atmospheric portion of the skylight changes from night to night; in the second place, the effect is highly dependent upon the brightness and the brightness distribution in the observed portions of the ZL. In the optimum case, the corrections can be used as average values for the Gegenschein.

The combined effect of the extinction and skylight which has been determined here also shows up in a similar manner in the brightness of points in the Milky Way. With increasing zenith distance, weak clouds in the Milky Way become brighter and brighter ones become weaker. In the former case, the amount of skylight in the measured total brightness is so great that its influence strongly predominates; in the case of the brightest clouds, on the other hand, we find practically pure extinction curves [3]. The contrast between the environment in each case naturally is less at the horizon and at high elevations.

(c) Deviation From the Point Opposite the Sun in Longitude

For the angle $\lambda - \odot$, which should amount to 180°, we obtained the following values (Table 4) by limiting ourselves to observations in which the hour angle of the Gegenschein was not greater than ±20°.

The agreement of the three average values is surprisingly good. It is still more remarkable that all three values deviate by more than 1° from the proper value of 180°. A. Searle [4] previously noted this phenomenon, but did not find it to be confirmed later in the Arequipa series. In fact, according to my analysis [1, p. 56] we have $\lambda - \odot = 180.42°$ and showing that the westerly deviation of the Gegenschein from the point opposite the Sun is at least not constant. Other series confirm this finding for both directions. It is not possible to give an explanation based on causes in observation technique. In fact, owing to the factor r^{-2} in the formula for the area brightness, an eccentricity of the ring would produce a gradient in the longitude, but the effect would cancel itself out in the course of the year; in addition, the conditions appear to be such that the majority of the observations were made in the vicinity of the /54 apsides, where the effect tends toward zero. Therefore, if we wish to consider that the deviations are real, we have as the sole possibility of an explanation, the assumption of nonuniform mass distribution along the ring; this is an assumption supported for all the general estimates that the zodiacal band is not always equally well visible between the Gegenschein and the principle light. For example, it was extraordinarily weak in March of 1933 despite favorable observation conditions (second expedition).

4. The Principle Light and the Zodiacal Band

(a) Statement of Purpose.

The principle purpose of the present paper is undoubtedly to derive the actual axis positions for various solar longitudes from the observations, so that the astrometric theory of the ZL given in [1] can be tested and corrected for this particular case and can even be improved. However there are a few minor goals in addition to this final goal, which should also be of importance for our understanding of the ZL.

The problem of the observation errors has remained unexplained until now. Mention has constantly been made of the effective extinction, so that calculations have been carried out using only the observations obtained at steep axis position to derive the results. However, no attempt has been made to determine the magnitude of the errors.

A special role is played in this regard by the latitude effect assumed by Schoenberg and Pich [5], in which the light axis (with the exception of the part belonging to the Gegenschein region) is assumed to be shifted toward the north in the Northern Hemisphere and to the south in the Southern Hemisphere. This problem can also be solved by means of the observations.

(b) The Effect of Extinction.

The available observations offer two possibilities for determining distortions of the light axis as a consequence of disturbing atmospheric influences:

1. Undisturbed observations are used to determine the true path of the light axis; this is then compared with observations made under less satisfactory geographical conditions. The distribution of the observations over 85° of latitude is suitable for this purpose.

2. Observations made at the same time of the year at Windhuk ($\phi = -22.6°$) with observations from the Northern Hemisphere, selecting those cases in which the light axis at Windhuk lies north of the solar vertical and south of the solar vertical at the northern reference point, and the angles of inclination at

the two places are not very different. The difference in the observed latitudes is the direct sum of the mutual deviations of the observed from the true light axis.

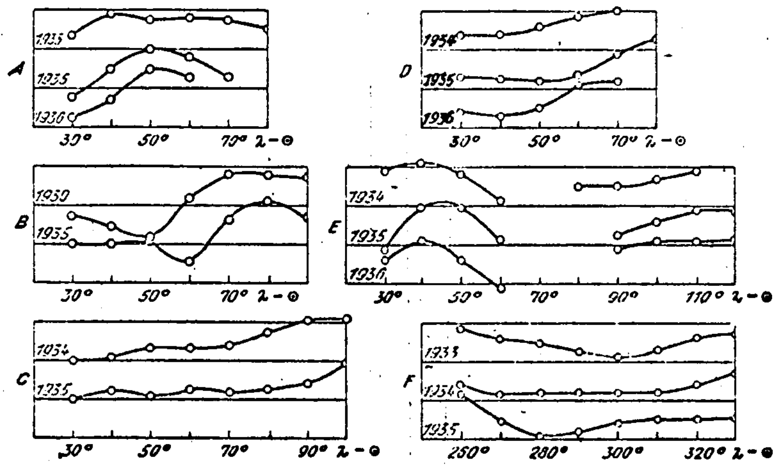


Fig. 4. Comparison of the Observations From Different Years at Nearly the Same Latitude. The Latitudes are Increased Five-Fold. The Zero Axis (Ecliptic) is Displaced Downward 2° for the Second and Third Annual Groups.

For the present investigation, it is immaterial whether the error caused by the influence of the atmosphere has a composite structure. In addition to pure extinction, the nocturnal sky-light and the twilight have an effect. Here, however, we are interested only in the total effect and in the latter only to the extent that it affects the ecliptic latitudes. /55

TABLE 5. DATA FOR FIGURE 4.

Group	Observation Date	n	Series	Group	Observation Date	n	Series
A	1933 Mar 20, 21, 23	33	2FF	D	1935 Mar 1, 2, 4, 6, 7	34	J
»	1935 Mar 22, 23, 24, 25, 26, 27	45	J	»	1936 Mar 10, 11, 12	23	J
»	1936 Mar 21, 23	16	J	E	1935 Mar 31, Apr 2, 3	22	J
B	1930 Feb 14, 15, 16, 19, 20	32	1FF	»	1936 Apr 8, 9	14	J
»	1935 Feb 23, 25, 27, 28	42	J	F	1933 Nov 15, 17, 19, 23	19	J
C	1934 Feb 1, 2, 8	22	J	»	1934 Nov 11, 14, 15, 16, 18	55	J
»	1935 Jan 26, Feb 2, 5	30	J	»	1935 Nov 5, 6, 25, 27	25	J
D	1934 Mar 5, 6, 8, 10, 11, 12	43	J				

Since the observation series of different origins overlap incompletely if at all, observations made in nearly the same solar longitude in different years must be compared. Hence, this means that we must first prove that this is actually the case, i.e., that the same position of the light axis recurs annually at the same solar longitude. Figure 4 lists six groups of comparable observations, so that the most probable position of the light axis can be found at 10° intervals from the plots of the individual values.

The single major contradiction in Figure 4, in Group B, can probably be explained as the disturbing effect of the group of stars $\delta\varepsilon\zeta$ Piscium. In general, the observations from different years do show frequent small differences, but there is a basic similarity of the pattern which often extends even to the details. Thus, if proof were still required that it is justifiable to combine observations from various years in determining the probable position of the axis, it would be provided by the result of this combination. The axis curves thus derived, listed in Tables 1 and 2, which are mentioned here somewhat in advance, also show occasional severe scattering and systematic omission of entire groups of observations; however, as far as the position of the axis is concerned, they show a very clear result without any significant distortion owing to the combination of observations from different years. It therefore appears that the axis position observed at a given solar longitude can be viewed as repeating itself annually.

We can now proceed to determine the extinction effect by the method described above. In spite of the considerable scope of the observation material, the possibility of using the method is somewhat limited because the observers were required to make their observations only under geographically favorable conditions and often failed to plot the axis under unfavorable conditions.

TABLE 6. COMPARISON OF AXIS LOCATIONS AT UNFAVORABLE POSITIONS ($z' > 15°$) WITH THE UNDISTURBED LIGHT AXIS (z' = ZD OF THE ECLIPTIC, A = EVENING, M = MORNING), WITH POSITIVE VALUES BEING OBTAINED WHEN THE SHIFT IS MADE IN THE DIRECTION OF THE EXTINCTION EFFECT.

Series	z'	A od. M	$\lambda - \odot$ 30° 330°	40° 320°	50° 310°	60° 300°	70° 290°	80° 280°	90° 270°	100° 260°	110° 250°	120° 240°	130° 230°	140° 220°	150° 210°	160° 200°	170° 190°
J	22°	A		+0°2	0°0	+0°6	+0°4	+0°3	0°0								
»	25	A	+1°0	+0.9	+0.7	+0.4	+0.1	0.0	−0.1								
»	29	A	+2.0	+1.5	+1.3	+0.3	−0.6	−0.8	−0.8								
»	21	M	+1.4	0.0	−0.4	−0.2	0.0	+0.6	+1.1	+1°0							
»	29	M	+2.4	+0.9	−0.2	+0.2	0.0	0.0	−0.2	−0.1	−0°3						
So	44	M				+0.7[1]							+1°0	+1°2	+1°2	+0°6	
Wi	21	M						+1.4	+0.5	+0.2	+0.3	+0°3	+0.4	+0.7			
So	28	A		+0.5			+0.4		−0.4								
Wi	27	A						+0.6	+0.5	+0.5	+0.2	0.0	+0.6		0.0	+0.5	+0°5
»	18	A	+0.6	+0.2	0.0	−0.1	−0.2	−0.2	−0.3								
»	35	A		+0.4	−0.1					−0.2	0.0						
Avg.	27°2		+1°48	+0°57	+0°18	+0°27	+0°01	+0°24	+0°03	+0°03	+0°05	—	—	—	—	—	—

[1]) $z' = 38°$.

As we can see from Table 7, the $\Sigma z'$ show no severe scattering with the exception of Groups 5 and 12, which are shifted in the opposite direction. Although there can be no approximately linear relationship between z' and Δ, the simple average values of the observed Δ will be used in the following. The method used here is still incomplete because it considers the slope z' of the ecliptic to the vertical of its horizon point, and not /56 the ZD of the observed point on the ZL axis, upon which Δ is naturally also dependent. If we can also assume that the observations of the principle light were always made at the same local time, the effects of different ZD on those parts of the axis which are near the horizon can become rather strong, while this is not the case for those which are located further toward the zenith. On the basis of the available observations, this effect can not be taken into account; however, it is not necessary so long as we agree to use observations up to approximately 35° elongation only if the ecliptic runs at a steep angle to the horizon.

It should be noted that the final values of Δ, which represent the influence of the atmosphere on the light axis, are given as simple values in Table 6, while Table 7 gives the sum of the values observed at z' = 23.8° and z' = 33.8°. Hence, the actual errors are approximately half as great as those in the values listed in the last column of Table 7.

TABLE 7. DIRECT COMPARISON OF OBSERVATIONS IN THE SOUTHERN AND THE NORTHERN HEMISPHERES (DIFFERENCES IN THE OBSERVED ECLIPTIC LATITUDES.

Group	series z'	A or M	$\lambda - \odot$ 30° 330°	40° 320°	50° 310°	60° 300°	70° 290°	80° 280°	90° 270°	100° 260°	110° 250°	$\Sigma z'$
1	Wi 23°, J 35	A		+2°.6	+2°.2	+1°.8	+1°.4	+1°.4	+1°.1	+1°.2	+1°.8	58°
2	Wi 32, J 31	M	+3°.0	+1.6	+0.8	−0.3	−0.1	+1.0	+1.4			63
3	Wi 39, J 22	M	+0.2	−0.4	−0.3	+0.6	+1.0	+0.2	+0.3	0.0		61
4	Wi 34, J 24	A	+1.4	+1.6	+1.4	+1.2	+1.6	+1.4	+1.9	+1.5		58
5	Wi 34, So 49	A			+2.6	+1.5	+1.1	+1.2	+1.5	+1.3		83
6	Wi 20, J 39	A		+2.2	+1.5	+0.6	0.0	0.0	+0.3			59
7	Wi 20, J 39	A		+2.0	+1.8	+0.4	−0.4	−1.0	−0.4	+0.3	+0.5	59
8	Wi 24, 2FF 30	A		+0.7	+0.6	+0.5	+0.9	+1.1	+1.3	+1.3		54
9	Wi 23, 2FF 29	A		+0.8	+0.9	+0.8	+1.1	+1.0	+1.6			52
10	Wi 23, J 33	A		+1.3	+1.4	+1.3	+0.2	−1.3	−0.9			56
11	Wi 23, J 30	A		+1.6	+1.4	+1.0	+0.2	−0.4	+0.5			53
12	Wi 21, J 21	M					+2.4	+2.0	+2.2	+0.3		42
13	Wi' 21, J 29	M					+1.7	+1.5	+1.2	−0.2	+0.3	50
avg. of smaller z'=23°.8, of larger z'=33.8			+1°.53 $\Delta_1 + \Delta_2$	+1°.40	+1°.30	+0°.85	+0°.85	+0°.62	+0°.92	+0°.71	+0°.87	57.5

TABLE 8. TABLE FOR CORRECTION OF OBSERVED LATITUDES OF THE POINTS ON THE AXIS FOR ATMOSPHERIC INFLUENCES.

z'	$\lambda - \odot$						
	30° 330°	40° 320°	50° 310°	60° 300°	70° 290°	80° 280°	90° 270°
16°	0°.2	0°.2	0°.1	0°.1	0°.1	0°.1	0°.1
18	0.2	0.2	0.2	0.1	0.1	0.1	0.1
20	0.3	0.3	0.2	0.2	0.1	0.1	0.1
22	0.4	0.4	0.3	0.2	0.2	0.2	0.2
24	0.4	0.4	0.3	0.3	0.2	0.2	0.2
26	0.5	0.5	0.4	0.3	0.3	0.3	0.3
28	0.6	0.5	0.4	0.4	0.3	0.3	0.3
30	0.7	0.6	0.5	0.4	0.4	0.3	0.3
32	0.8	0.7	0.6	0.5	0.4	0.4	0.4
34	0.9	0.7	0.6	0.5	0.4	0.4	0.4

In view of the greater importance of the values in Table 7, Table 8 has been drawn up and can be used to correct the observed axis points for the influence of the atmosphere. For the range $80° < (\lambda - \odot) < 280°$, the figures can be viewed as constant. In this respect the results show good agreement with the results for the Gegenschein, where the error reaches 0.75° at approximately 40° ZD.

It hardly needs to be mentioned that in this discussion the atmospheric disturbance of the light axis in ecliptic latitude is considered as the only factor influencing observation.

Generally speaking, it can be said that the errors produced by the atmosphere are less than expected. As long as the values located near the lower left corner of Table 8 and those in which z' exceeds 35° are eliminated, the errors are small relative to the ecliptic latitudes of the axis points which are to be determined and amount to 3° at most. When the observations are distributed over the Northern and Southern Hemispheres of the Earth, compensation is also involved. The remaining errors in the values in Table 8 are therefore of no importance for the total picture. /57

(c) The Hypothetical Latitude Effect.

Schoenberg and Pich, on the basis of the observations which they evaluated and those made by the Breslau observers in Windhuk (ϕ = - 22.6°) and those of Jones (various ϕ, + 22° on the average), found that the line of symmetry of the entire ZL axis (with the exception of the Gegenschein region) deviates from the ecliptic southward in the Southern Hemisphere and northward in the Northern Hemisphere. The total amount of this "negative parallax" is approximately 2° with a difference of 45° between the latitudes of the observation points. The authors considered this phenomenon to be a sign that the ZL is always subject to atmospheric dimming, caused by the highly extensive atmosphere of the Earth. The large axis of this atmosphere would then correspond roughly to the extension of the polar axis of the Earth, thus producing brightness gradients which are positive toward the pole and disturb the ZL axis in a similar manner, as we can see in Section 3b of this paper with regard to the skylight and the Gegenschein.

The observations which we have presented show unambiguously that a latitudinal effect of such magnitude does not exist. Neither in Table 6 nor Table 7 are the differences so great that a value of 2° could be attributed to the extinction effect. In particular, a direct comparison of the northern and southern observations in Table 8 would show the effect under all conditions, since the difference in latitude (Windhuk-Jerusalem equals 54.4°) still greater than in the results given by Schoenberg and Pich. However, in view of the experiences with the Gegenschein, there is not the slightest reason for interpreting these results as being caused by anything other than extinction.

We could be satisfied with this finding if it were not interesting to determine the manner in which the latitudinal effect is produced. We believe we can prove that it is primarily a selection effect which is involved.

According to our earlier investigations, the entire mass of the ZL is divided into two mutually related parts, the inner portion with a maximum density in the vicinity of the orbit of Venus and the definitely annular outer portion between the orbits of Mars and Jupiter. The outer portion is aligned with the plane of Jupiter's orbit, with the ascending node at 100°. The plane of symmetry of the inner portion is not flat, but can be viewed as such here for the sake of simplicity; the longitude of its ascending node is approximately 50°. The principle light is caused primarily by the inner portion.

If we assume this state of affairs to be as represented, we will see that the Earth lies on the node line of the inner portion at a solar longitude of 50° (May 11), so that the light axis always has positive latitudes to the east of the Sun and negative latitudes to the west of the Sun. Six months later, at $\odot$ = 230° (Nov. 13), the negative latitudes are located to the east of the Sun and the positive latitudes are located to the west of the Sun; in other words, in late Spring (in the Northern Hemisphere) the axis of the twilight lies north of the ecliptic while in late Autumn the axis of the dawn light lies north of the ecliptic. In addition, it is always those parts of the axis which lie north of the ecliptic which are located in the favorable position relative to the horizon. The situation in the Southern Hemisphere is exactly the same with respect to the portion of the axis with southern ecliptic latitudes. If, like Jones, for example, we discard all the observations in which the inclination of the ecliptic to the vertical exceeds a certain value, we can see that in the Northern Hemisphere it is primarily the northern ecliptic latitudes of the light axis which are observed and predominantly southern ecliptic latitudes which are observed in the Southern Hemisphere. On the other hand, if we do not discard these observations and do not correct for them (which has never been attempted before), we will be taking the effect of extinction into account, which once again displaces the light axis to the north in the Northern Hemisphere and to the south in the Southern Hemisphere. This is what causes the latitudinal effect which we have been discussing.

On the basis of their Figure III, Schoenberg and Pich believed they have proven that extinction has only a very slight influence on the Windhuk observations. However, this conclusion /58 is not valid because in that Figure λ is used as an argument and the latitudes found for each λ are averaged over the entire year. In the course of a year, a point with the longitude λ passes through all parts of the ZL. Depending on its position on the node line of the ZL, various average latitudes will be obtained which can be used only with qualifications as indications of the mass structure. An obscuring effect is also produced by various factors such as the complex structure of the ZL, the various distances of the effective parts from the Earth, as well as the selection effects caused by the fact that the Sun, the unsatisfactory position relative to the horizon and (to a certain extent) the Milky Way as well, can considerably limit the time during which the point can be observed. If we proceed in the manner described in Figure III [5], the observations distorted by extinction will be confused with those which have not been distorted and the detection of the disturbances is considerably complicated. This is especially true at the longitudes 0° and 180° which were observed best. If 0° is located on the eastern horizon relative to Windhuk, then the ecliptic runs perpendicular to the horizon ($z' = 0°$); if 0° is located on the western horizon, $z' = 46°$, i.e., as unsatisfactory as possible. The same is true for the conditions at 180°. In the range from $\lambda = 0°$ to $180°$, therefore, observations at the most favorable and least favorable positions are included in the annual average for the latitudes. If the most favorable positions are much more numerous than the least favorable ones, as is usually the case, even severe distortions caused by extinction can be reduced to a quite small value in the annual averages.

On the other hand, Figure IV [5], in which the annual course of points with a certain solar elongation are shown, indicates definite influences of extinction. The Curves A and M plotted for the twilight and dawn are valid for points with 65° easterly and westerly elongation. If we once again assume that the astrometric theory of the ZL which we have developed is valid, the apparent ascending nodes for the twilight would lie at about $\lambda = 10°$ and those for the dawn would lie at about $\lambda = 105°$ (on the basis of the diagram in Figure 25 on p. 91 in [1]). It is particularly the curve of the dawn which contradicts this expectation, because it is precisely where the observations must be carried out between 110° and 230° northern latitude that the maximum southern deviations occur. To explain this contradiction, as well as another, less significant one in the curve of the twilight at $\lambda = 40°$, we must determine at what times the twilight and dawn in Windhuk are so unfavorably located that significant extinction effects can be expected. We will assume that this is the case when the angle between the ecliptic and the vertical to its base increases to more than 30° and also that the observations of the principle light were made at a ZD of the Sun of about 110°; we then find that the beginning and end of the most unfavorable time for the twilight are December 18 and May 16 (solar longitudes 265° and 54°), while those for the dawn are July 31 and January 1 (solar longitudes 127° and 280°). In Figure IV in Schoenberg and Pich's paper, these values correspond for the twilight to the range λ = 330° to 119°, and 62° to 215° for the dawn. The maximum southern deviations in both cases lie precisely in these unfavorable regions and can therefore be considered with a high degree of probability to be caused by extinction. Perhaps the relatively high significance of the extinction effects can be attributed to the use of outlines rather than direct observations of axis points.

In the curve based on observations by Jones [5, p. 12, Fig. III], the limitation to steep positions of the ecliptic precludes a significant extinction effect; in this case, the predominance of observations from the Northern Hemisphere and their combination with other observations due to the averaging can be viewed as the only cause of the effect. In a previous paper, we made the following remark regarding Jones' observation series [1, p. 73]: "The predominance of northern tropical and subtropical observation sites is so great that the abovementioned conditions for the twilight are fulfilled only in the Spring and for the dawn mostly in the Autumn". A selection of this type, in view of the findings mentioned above, must necessarily lead to a displacement of the axis of symmetry northward.

In the preceding, we have assumed the astrometric theory of the ZL as given. There is a possibility that it is only one possible explanation and that the other possibility might lie in an interpretation of the phenomenon as a latitudinal effect. However, this hypothesis has already been refuted by the data in Tables 6 and 7.

Even if the latitudinal effect suggested by Schoenberg and Pich could not be confirmed, we cannot completely reject the idea. It is probable that the brightness of the night sky from the Equator toward the pole decreases, and it is highly likely that this can produce a displacement of the axis of the ZL, though very slightly.

5. The Path of the Light Axis at all Times of Year /59

After all the prerequisites have been satisfied, the light axis itself can be plotted. In order to permit an estimate of the accuracy attained, we have avoided doing any averaging and have plotted the individual observed points. The year has been divided into segments of 30° solar longitude; to the extent that the density of the series permitted it, regions of 20° were used. As a result, the 14 representations of the light axis given in Tables 1 and 2 were obtained.

We will consider as undisturbed, all observations in which the ecliptic forms an angle of at least 15° with the vertical to its horizon point. We will also include the observations in which this angle z' was 15 to 30°, after correction according to Table 8. At those points where the light axis was insufficiently determined by observations, we have entered corrected observations with values of z' as high as 35°, excepting however the portions of the light axis near the Sun (i.e., near the horizon) (Table 9).

TABLE 9. OBSERVATIONS USED FOR PLOTTING THE LIGHT AXIS

Type	z'	Number
a) uncorrected	0° to 15°	2652
b) corrected	15° to 30°	1298
c) corrected	30° to 35	270
TOTAL		4220

Out of a total number of 4934 observations, 86% were used in plotting the axis. The unused remainder were almost exclusively observations with $z' > 35°$. In the case of one observer, however, a few partial series had to be discarded because of evident distortion which this observer himself mentioned in his remarks. However, this limitation affects one of the most densely populated positions in the entire series, twilight around the time shortly after the vernal equinox, and is therefore insignificant for the result.

The plotting of the points on the graph in Tables 1 and 2 shows occasional systematic deviations of individual groups; it could not be otherwise for such painstaking observations, whose selection was so highly dependent on the personal inclination of the observer as well as on a number of terrestrial and extra-terrestrial circumstances which are rather hard to define (twilight, luminous clouds, the Milky Way, bright stars, groups of weak stars, etc.). Nevertheless, the general impression is that the axis has been determined satisfactorily for all regions and for all seasons. In general, the points of intersection with the Milky Way have been omitted. The resultant gaps are wide when they happen to coincide with the weakest portions of the ZL, but are compensated for in the brightest parts of the principle light. As an example of such a disturbance caused by stars, such as undoubtedly occurs at other points as well, without the averaging over 30° or 20° of solar longitude being able to cancel it out, we can use the steep slope of the light axis from the southern to the northern latitudes and the increased scatter of the points which first shows up in Group 6 at 260° eastern elongation and can be followed up to Group 9, where it lies at 210° eastern elongation. The maximum northern latitude falls at about 60° ecliptic longitude, exactly at the longitude of the Pleiades, whose ecliptic latitude is + 4°. Perhaps it is not the Pleiades alone which cause the disturbance but also the weak portions of the Milky Way which extend from the northeast to the ecliptic in this region. This same position is found again in Group 12 at $\lambda - \odot = 110°$, but in this case somewhat more severe scattering is found. The degree of disturbance increases as weaker portions of the ZL are examined. Mention should be made in this respect of the phenomenon (frequently very clear) that the middle of the Gegenschein lies closer to the ecliptic than the adjacent portions of the zodiacal band would lead us to expect. As pointed out in Section 3 of this paper, this is an effect of the phase law.

Explanation of Tables 1 and 2

The argument of the tables is the eastern elongation of the Sun, whose position is assumed to lie at 0° and 360°. We will assume that the twilight ZL is located on the right-hand side (20° to 90°), the dawn ZL is located on the left side (270° to 340°), and the Gegenschein is located in the center (180°). The axis of the abscissa is the ecliptic. The latitudes are enlarged fivefold relative to the longitudes. Near the left-hand edge, there /60 is an indication of the region of solar longitudes for which the representation of the axis is valid. Observations made in the Northern Hemisphere are represented by dots, while observations in the Southern Hemisphere are represented by circles. The gaps in the tables are produced by the Milky Way.

6. Constants of the Plane of Symmetry of the Zodiacal Light

(a) General Remarks.

The axis points for different western and eastern elongations obtained from Tables 1 and 2 are used as the bases for the following calculations. Instead of calculating an average, which leaves doubt as to which longitudinal difference the observations can be limited, we have estimated the latitudes of the axis points in tenths of degrees, taking into account both the distribution of the individual observations within ± 5° difference in longitude with respect to the desired point as well as the path of the axis in the adjacent sections.

In our first paper [1, p. 49], we made a detailed study of the factors affecting the position of the ZL and developed the astrometric theory. For the sake of economy, we will avoid repetition and refer the reader to that paper for all details including the symbols. We will make explanatory comments only where it is required for understanding. In order to make it easier for the reader to find his way through the earlier discussion, we will provide appropriate directions in brackets.

(b) The Position of the Outer Ring.

The annual path of the Gegenschein has already been determined. Here we will investigate two additional elongations which belong to the zodiacal band: $e = \pm 160°$ and $e = \pm 135°$ (+ equals easterly, - equals westerly with respect to the Sun). The latter position corresponds to the weakest point in the zodiacal band, and the former lies very close to the Gegenschein; in both cases, we can reasonably expect to determine the appearance of the outer ring accurately on the basis of the effects of the portion of the inner section located outside the Earth's orbit (Table 10).

TABLE 10. VALUES OF λ AND β FOR THE ZODIACAL BAND ACCORDING TO TABLES 1 AND 2

$e = -160°$		+160°		−135°		+135°	
λ	β	λ	β	λ	β	λ	β
0°5	−1°9	0°5	−1°4	0°5	−2°2	0°5	−1°9
20.5	−1.4	25.5	−1.0	25.5	−0.8	30.5	−1.0
40.5	−0.6	55.5	0.0	150.5	+0.5	145.5	+1.3
65.5	0.0	85.5	0.0	180.5	+2.0	165.5	+1.9
95.5	+0.6	115.5	+1.0	210.5	+2.6	185.5	+2.2
125.5	+0.2	145.5	+1.0	235.5	+2.1	210.5	+1.0
155.5	+1.2	170.5	+2.2	330.5	−2.0	335.5	−2.0
185.5	+2.6	190.5	+2.2				
210.5	+2.7	210.5	+2.1				
230.5	+0.8	235.5	+1.1				
250.5	0.0	265.5	+0.6				
275.5	+0.6	295.5	+0.7				
305.5	−0.6	320.5	−0.5				
335.5	−1.7	340.5	−1.6				

TABLE 11. RESULTS OF COMPENSATION FOR NODES AND APPARENT SLOPE OF THE OUTER RING

e	Ω'	avg. error	β	i'	avg. error	i' avg. value	avg. error	Ω
180°	111°5	±20°4	+	2.48	±0.58	—	—	111°5
			−	1.93	±0.18			
−160	93.0	± 6.9	+	2.08	±0.38	1°67	±0°24	96.0
			−	1.33	±0.29			
+160	99.1	± 7.3	+	2.00	±0.16	1.58	±0.23	
			−	1.08	±0.24			
−135	118.4	±11.0	+	1.95	—	1.85	±0.50	95.5
			−	1.71	—			
+135	72.6	± 8.3	+	1.79	—	1.84	±0.15	
			−	1.87	—			

At e = ± 160°, the points at which the band could not be observed because of the Milky Way have been filled in on the basis of the path of the light axis in the vicinity of these points; at e = ± 135°, this could not be done because of the disturbances of a further region by the Milky Way, which were much stronger here.

Table 11 contains data on the comparison, including the apparent nodes Ω', as obtained directly from the observations, the apparent slope i' initially separated for positive and negative latitudes, then the average value obtained when using all observations, and finally the true nodes Ω, corresponding to the midline of the directions of the true apparent nodes. The partial values for i' for positive and negative latitudes were determined at e = ± 160°, omitting those values which lie within ± 30° of the nodes.

For the node longitude, we obtain as the average of the three values Ω = 101.0° ± 5.3°, which is in very close approximation to the nodes of the orbit of Jupiter, 99.56° (1925.0).

With the highly probable assumption that the plane of the rings corresponds with that of Jupiter's orbit, we can (as was pointed out earlier [1, p. 58]) explain the surplus of observed apparent inclinations relative to the inclination of Jupiter's orbit as a parallax of the ring, such as would be produced in observations from the Sun and from the Earth, and use them to determine the distance of the ring. /61

TABLE 12. PARALLAXES AND RADII OF THE OUTER RING

	i'	$i'-i$	av.er	r
β NORTH	2.15°	0.84°	±0.09°	2.56 AE
β SOUTH	1.64°	0.33°	±0.18°	4.99 AE

The values given above for i' are not strictly comparable, since those observed at e = 160° and 135° at the same distance from the Earth Δ must be reduced, since it corresponds to the Gegenschein. The reductions, for which a knowledge of the approximate distance of the ring is required, are slight and amount to about 1-2% at 160° and 7-10% at 135°. This gives us the values in Table 12.

As we can see, there is a very noticeable and improbably large eccentricity of the ring. However, as the average error shows, the second value of r was determined with considerable inaccuracy. If we use a value of i' = 1.93° for it, in accordance with the observations of the Gegenschein, we obtain r = 3.12; if we enlarge the average obtained for southern β by the amount of the average error to 1.82°, we obtain r = 3.425. Hence, if we use r = 3.4 AE for the aphelion distance of the ring, we will not be contradicting the observations and will avoid lack of agreement in another respect. The orbital elements of the ring can then be assumed to be as follows:

$$\Omega = 101.0^\circ \quad i = 1.31^\circ \quad \omega = 90^\circ \quad a = 2.98 \text{ AE} \quad e = 0.11.$$

As has been mentioned earlier [1, p. 68], there are several factors which contradict assumption of a pronounced eccentricity of the ring on photometric grounds: owing to the factor r^{-2} in the brightness formula, the more distant portions would have to appear weaker than the closer ones, but thus far neither the problem of observation nor that of an approximate compensation by the effects of the density rule have been sufficiently explained. In any case, however, it is definite that the southern ecliptic latitudes of the ring axis are smaller than the corresponding northern ones; and secondly, the assumption of an eccentricity of the ring would be the best explanation for this finding. The observations also make it possible to reduce the eccentricity somewhat below the value assumed, without creating inadmissible contradictions. We have also examined the problem of disturbance caused by the Milky Way, but reached the conclusion that the flight effect which is noticed in this regard is insignificant for the result desired here. Hence, there are no serious objections to the explanation given.

As was also pointed out earlier, the aphelion of the ring would be turned toward the perihelion of Jupiter. Probably the distance of the perihelion from the node is not far different from 90°, since a more marked deviation would have to show up in the form of a deformation of the annual sinusoidal motion. The value for Jupiter is $\omega = 273°$.

(c) Plane of Symmetry of the Inner Portion.

In Table 13, we have assembled the directly calculated values corresponding to the table given in [1, p. 83], with the addition of the weights p_1 of the partial values of i, which will be explained later.

We have shown the bases for these calculations in Figure 5, in order to show the extent to which the annual sinusoidal motions of the individual points that were assumed in the theory are actually found in nature. The latitudes of the axis points taken from Tables 1 and 2 are freed of the influence of the outer ring according to [1, Formula 52].

In addition, the constants of the plane of symmetry Ω and i were calculated for each elongation according to [1, Formula 43], as well as the distances of the corresponding principal point from the Earth and Sun Δ_m and r_m according to [1, Formula 53]. Figure 6 shows the method.

Table 14 contains the results.

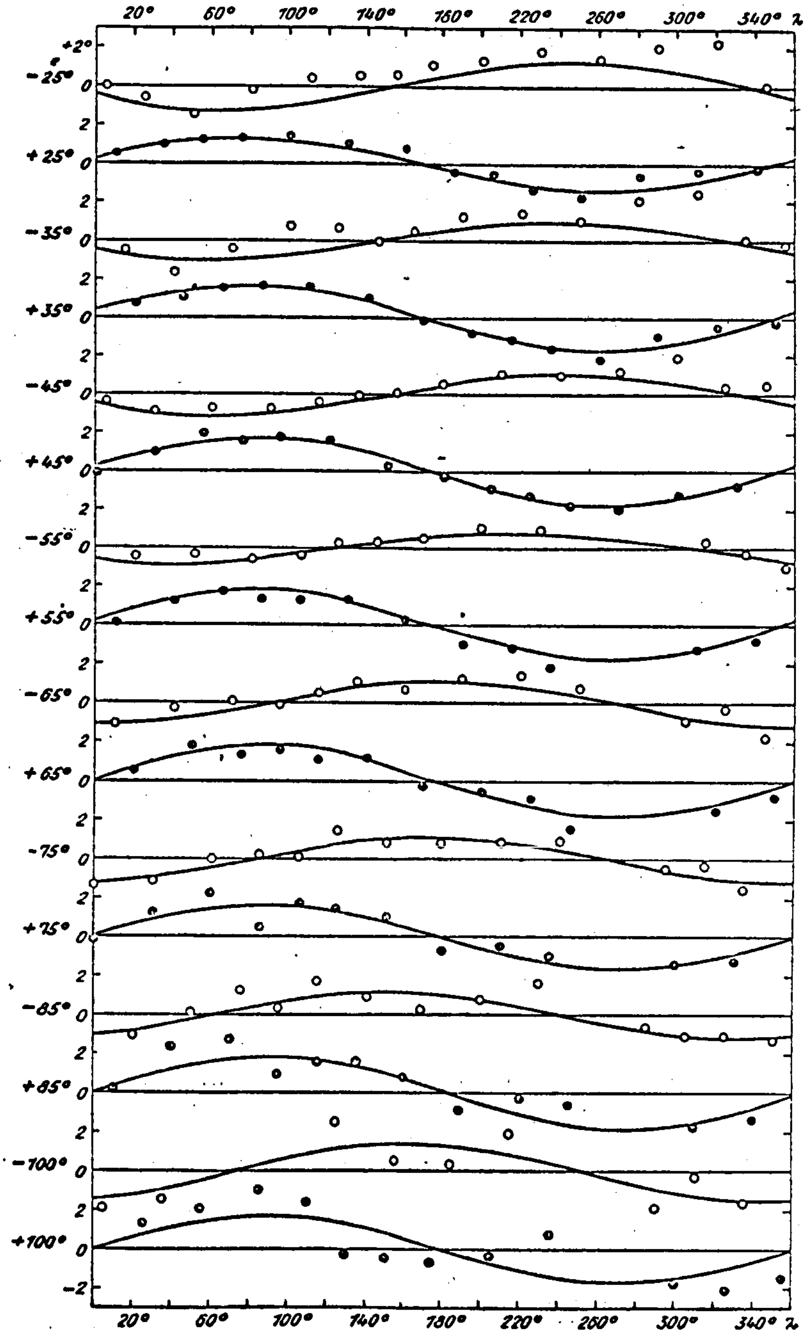


Fig. 5. Annual Pattern of the Ecliptic Latitudes of Axis Points at Different Elongations *e* (- Equals Western, + Equals Eastern). Observed Values and Balanced Sine Curves. The Observed Values Were Taken From Tables 1 and 2 and Corrected for the Influence of the Outer Ring.

TABLE 13. GEOCENTRIC NODES AND INCLINATIONS.

e	Ω'	av.er.	i'	av.er.	p_i
−25°	157°2	±8°9	1°25	±0°38	0.92
+25	350.1	±5.3	1.34	±0.13	0.99
−35	148.2	±23.3	1.00	±0.38	0.85
+35	349.6	±3.8	1.64	±0.11	0.98
−45	152.7	±13.7	1.12	±0.24	0.89
+45	352.9	±3.1	1.75	±0.09	0.99
−55	126.5	±7.2	0.78	±0.14	0.59
+55	352.8	±4.4	1.73	±0.14	0.99
−65	90.7	±8.9	1.12	±0.18	0.01
+65	358.3	±6.5	1.82	±0.19	1.00
−75	85.1	±8.0	1.16	±0.16	0.09
+75	359.4	±8.4	1.61	±0.24	1.00
−85	62.7	±12.8	1.17	±0.26	0.46
+85	3.2	±10.1	1.84	±0.33	1.00
−100	75.6	±21.3	1.51	±0.42	0.25
+100	0.8	±14.2	1.70	±0.54	1.00

A comment should be made regarding the calculation of i from the two partial values. In Table 13, correspondingly equal eastern and western elongations have been listed for i' value pairs. One might expect that equal values would be obtained for the eastern and western sides of the Sun; in addition, the values belonging to positive e are always the larger ones. The same phenomenon was found in the observations of Jones [1, Table on p. 84], except that here the values belonging to negative e are the larger ones. We believe that this affect can definitely be attributed to disturbance caused by the Milky Way, and will give further details in this regard in the next section. Here we will merely point out that the weight distribution used in [1, p. 84] has not been corrected for the average errors. The reader is likewise referred to the next section for the weight distribution which was actually selected. /63

The change in the weight distribution necesitates a partial recalculation of the survey given in [1, p. 84]. All of the corresponding values have been listed in Table 15. In view of the high average error for the partial value, which has the greater weight according to the method used here, each two values were assumed to have the same weight at elongations 105° and 120°. The elements were adjusted to the equinox 1925.0.

For comparison, we have listed here the elements of the inner planets which are involved, corrected to 1925.0:

	Ω	i
Mercury	47.47°	7.00°
Venus	76.07°	3.39°
Mars	49.05°	1.85°

An examination of the results confirms the conclusion drawn earlier: "the plane of symmetry of the internal zodiacal portion is not a plane; its position is determined instead by the orbits of the inner planets".

The node longitudes show a retrogressive tendency with increasing r according to the new calculations. However, up to r = 120° no influence of Jupiter can be detected, to the extent that

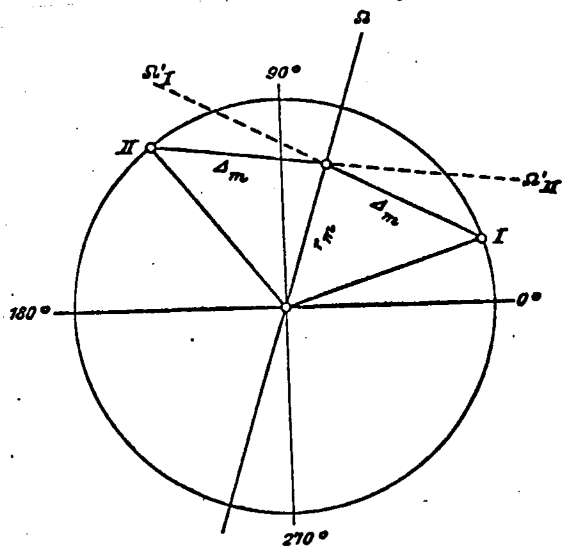


Fig. 6. Determination of the position of the principal point for e = 45° (Tables 13 and 14).

	I	II
$e=$	$-45°$	$+45°$
$\Omega'=$	152.7	352.9
$\odot=$	197.7	307.9
$\Upsilon_\odot=$	17.7	127.9

$\Omega = 72°8$ $\quad \Delta_m = 0.833$ $\quad r_m = 0.718$

TABLE 14. CONSTANTS OF THE PLANE OF SYMMETRY OF THE INTERNAL PORTION

e	$\Omega_{1925.0}$	av.er.	Δ_m	r_m	i'	$i_{1925.0}$
25°	73°7	± 5°2	0.954	0.425	1°30	2°92
35	68.9	±11.8	0.928	0.584	1.34	2.13
45	72.8	± 7.0	0.833	0.718	1.45	1.68
55	59.6	± 4.2	0.923	0.891	1.38	1.43
65	44.5	± 5.5	1.291	1.256	1.81	1.86
75	42.3	± 5.8	1.300	1.421	1.57	1.44
85	32.9	± 8.2	1.831	2.008	1.63	1.49
100	38.2	±12.8	1.462	1.621	1.66	1.50

TABLE 15. CONSTANTS OF THE PLANE OF SYMMETRY OF THE INNER PORTION ACCORDING TO THE OBSERVATIONS OF JONES

e	$\Omega_{1925.0}$	av.er.	Δ_m	r_m	i'	$i_{1925.0}$
30°	63°4	±10°5	1.013	0.521	2°59	5°04
45	57.7	± 9.5	1.102	0.810	1.61	2.19
60	43.8	± 8.5	0.941	0.972	1.17	1.04
75	47.5	± 9.4	0.974	1.202	1.52	1.23
90	48.6	±17.1	0.617	1.175	1.98	1.04
105	46.0	±17.5	0.509	1.234	3.14	1.30
120	70.7	±15.3	0.952	1.690	3.43	1.93

the data involved are admissible. On the other hand, the value for e = 120° in the observations of Jones indicated a considerable increase in the node longitude. The entire course of the node longitudes is very much in agreement with the assumption that it is primarily the orbits of Venus and Mars which have the important influence here.

The inclinations are listed in Figure 7 together with the radius vectors of the planets, with a five-fold increase of the angle, exactly as in Figure 23 on [1, p. 86]. From the data in Jones' observations, we see that the changed weight distribution /64 has a noticeable influence only on the value for r = 0.8, and that in fact it now shows a better agreement with the new data. A contradiction shows up at the smallest distances, where the single value obtained on the basis of Jones' observations shows a greater inclination. Here, however, the new values should be preffered because they are much more reliable.

The total impression can be summed up as follows: between r = 0.4 and r = 1.0, the position of the plane of symmetry is determined by the orbits of Venus and Earth. An effect caused by

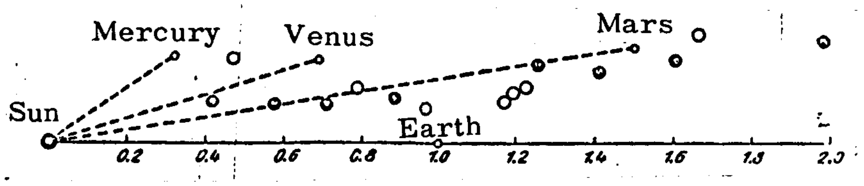


Fig. 7. Plane of Symmetry of the Zodiacal Light and Orbital Planes of the Planets Near the Sun (Angles Increased Five Times).

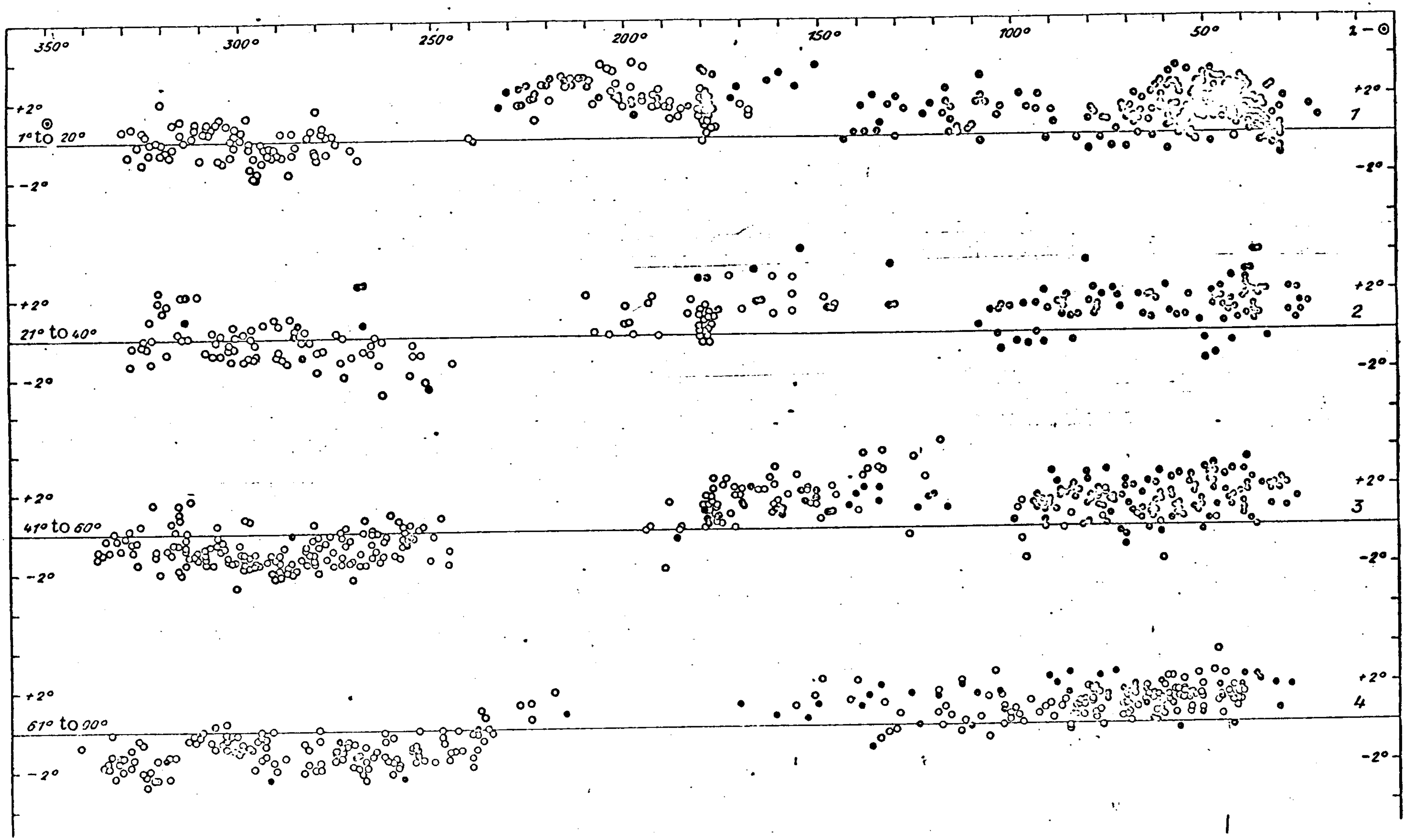
Mercury cannot be determined exactly. At $r > 1.2$, the influence of Mars has a momentarily greater effect ($i = 1.85°$), which is somewhat surprising in view of its low mass. However, it should be noted that its effect is supported by that of Jupiter ($i = 1.31°$). If we use only the nine newer values as a basis, we might be inclined to represent them by a plane with $i = 1.5°$, but we must always remember that even then there would be a deviation toward a greater inclination in the vicinity of the orbit of Venus and a deviation toward a smaller inclination in the vicinity of the Earth's orbit.

The jump between $e = 55°$ and $65°$ from $r = 0.89$ to $r = 1.26$ is striking in Table 14 and Figure 7. An interesting consequence of this leads us to discuss briefly the manner in which the data were obtained. At approximately $e = 60°$, the photometric principal point crosses the Earth's orbit. Here and at greater elongations, we observed the light axis created by the total effect of the portions of the inner and outer ring which are the same order of magnitude. These results are later corrected mathematically for the influence of the outer ring, whose elements are well known from observations in the vicinity of the point opposite the Sun, and we can expect that this will give us a true picture of the effect of the part of the inner portion which lies outside the Earth's orbit. The surprising thing is that this gives us a usable light axis which also apparently corresponds to reality. Since the space between the Earth's orbit and the inner edge of the outer ring cannot be empty, it is a fact which we have often pointed out in our studies. However, if we ascribe these masses to the inner portion (as is indicated by the elements of its plane of symmetry) and assume that the density decreases steadily from the Earth's orbit outward to the inner edge of the outer portion, no effect of the type observed could possibly occur in view of the very precisely obtained photometric data used in formulating the through-portion theory. Rather, we must assume that there is a minimum density in the vicinity of the Earth's orbit and that there is an increase in the direction of Mars' orbit. This would mean that the inner portion has no uniform structure from the photometric point of view and that we actually must differentiate between three portions.

It should be noted in this regard that in these studies the inner portion is defined photometrically and not physically; to it are ascribed all the effects which remain when we disregard the effect of the outer portion which is assumed to be related to Jupiter's orbit. On the contrary, it is much more likely that those parts which have been detected between the orbits of Earth and Mars have already been assigned to the outer portion, so that properties have been assigned to its plane of symmetry similar to those of the inner portion in the space between the orbits of Venus and Earth; this appears to prove that the outer portion extends close to the Earth's orbit in analogy to the newest theories regarding the system of the lesser planets.

On the basis of a survey presented on p. 80 in [1], the light effects of the outer portion (assumed directed toward the orbit of Jupiter) has a ratio in the Gegenschein region to the mass ascribed to the inner portion (lying outside the Earth's orbit) of roughly 2 to 1. This ratio has been determined mainly on the basis of the density distribution outside the Earth's orbit and is very indefinite; this is not important, however, because it was done only to free the light axis of the brighter portion of the ZL from /65 the influence of the outer portion. Now that all numerical values have been confirmed with a greater degree of accuracy, the question arises of the disturbance of the observations of the Gegenschein by masses which are not associated with the orbit of Jupiter but which lie outside the Earth's orbit. Answering this question will lead us beyond the limits drawn by the nature and accuracy of the available observations. However, the following can be said: the relationship of the outer ring to the orbit of Jupiter is a valuable finding; the fact that the observations of the Gegenschein and the weakest portions of the zodiacal band do not contradict this finding, and particularly show no sharp deviation of the node longitudes, can be viewed as an indication that the disturbances caused by the light effect of other masses located outside the Earth's orbit cannot be very strong. This finding could be explained if we were certain that the theory of the photometric principal points [1, p. 74] could also be applied to the Gegenschein region. In the case of elongations in the vicinity of 90 to 120°, this is clearly not so, for otherwise it would not have been possible to isolate mathematically a portion which can be assigned to the orbit of Mars. However, it is possible or even probable that at greater elongations the prerequisites for the use of the principal states in [1] will be satisfactorily fulfilled. It may be that between the Earth's orbit and that of Mars there is first a zone of approximately constant density which precedes a rise in density to that of the true outer ring. In the direction of the Gegenschein region, such a mass distribution would also produce hardly any disturbances because the drop in brightness at ecliptic latitudes would be reduced by the shorter distance from the Earth.

TABLE 1. C. HOFFMEISTER: AXIS OF THE ZODIACAL LIGHT AND CONSTANTS OF ITS PLANES OF SYMMETRY



Axis Points Observed for the Zodiacal Light in Fourteen Series Taken Throughout the Year. Abscissa: Elongation to the East of the Sun; Ordinate: Ecliptic Latitude; at the Left: Limits of the Solar Longitudes for Each Group. o = Observed from the Northern Hemisphere. ● = Observed from the Southern Hemisphere.

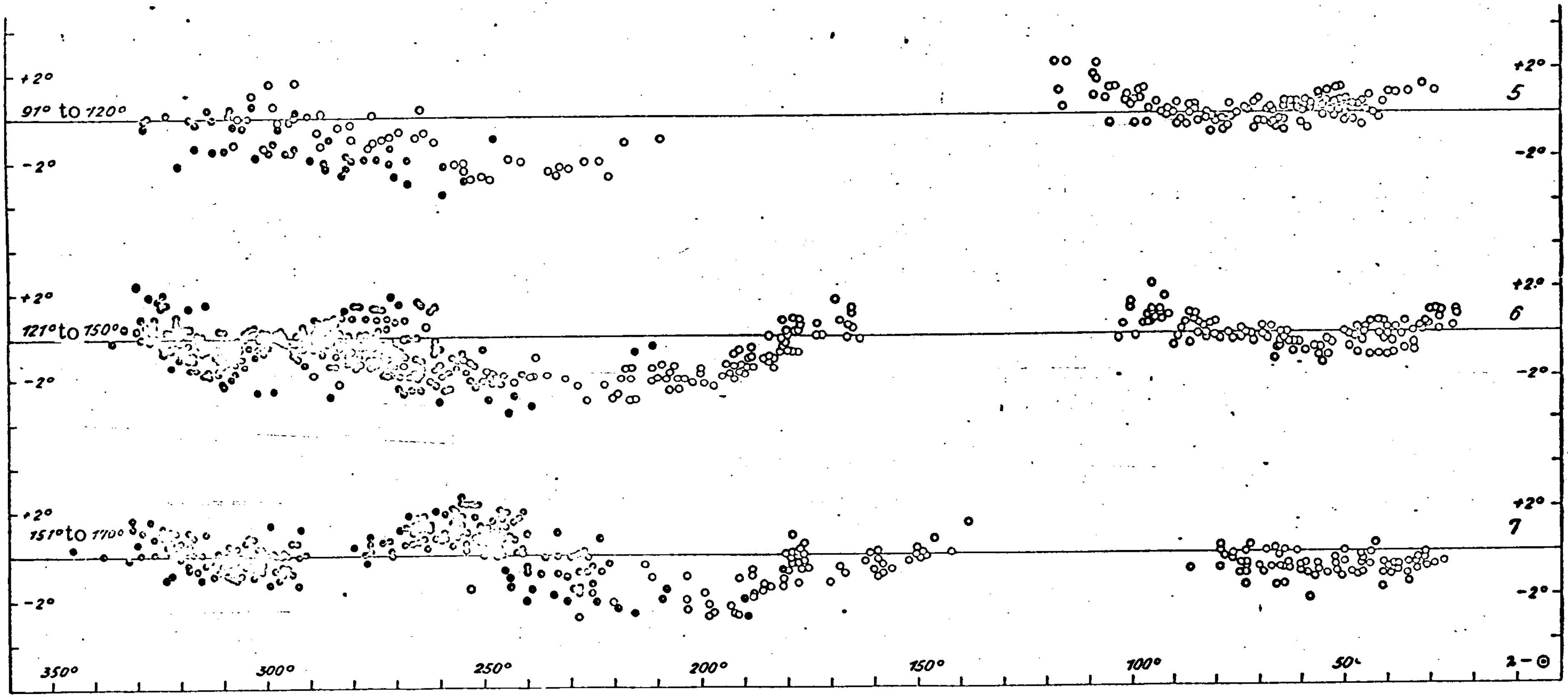
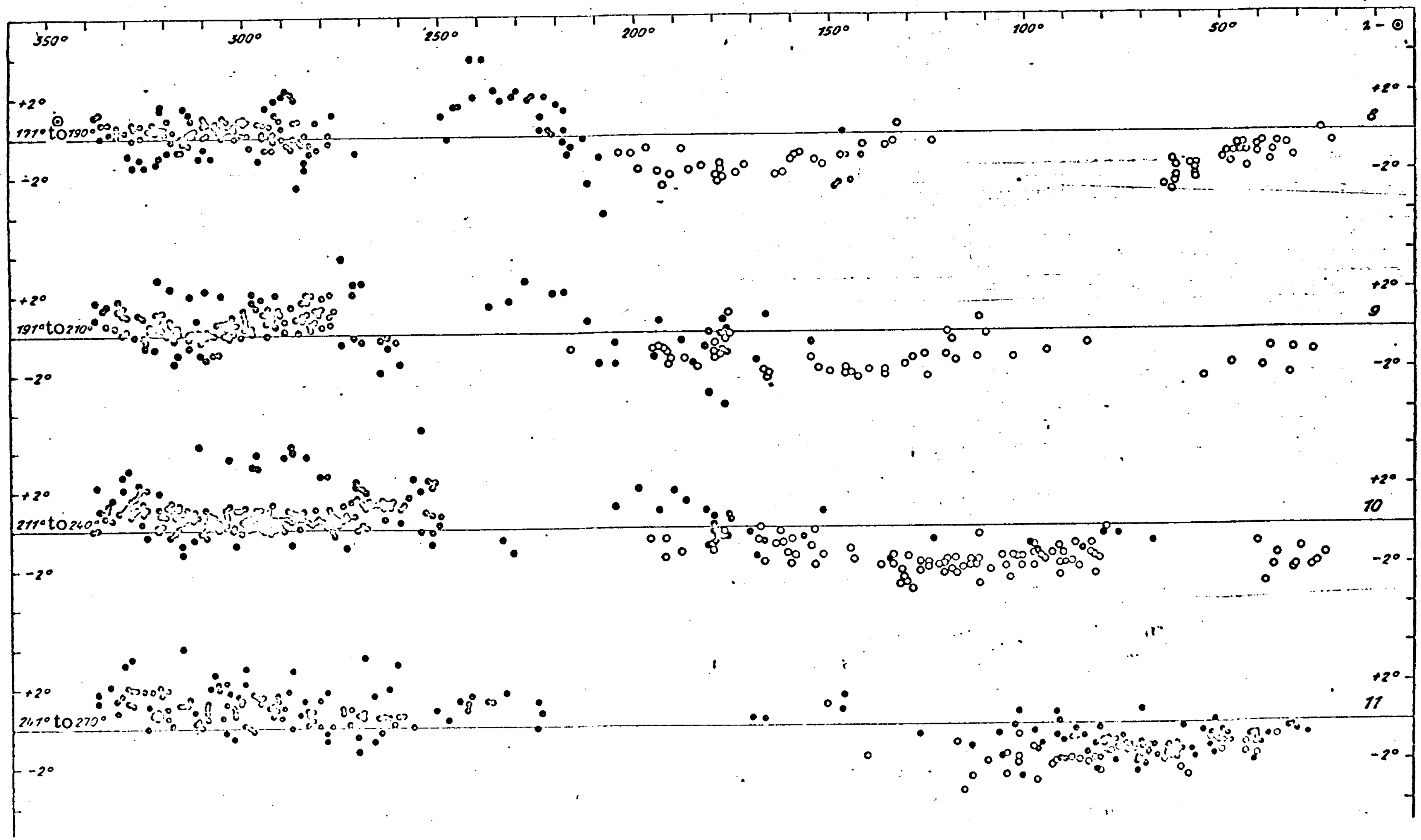
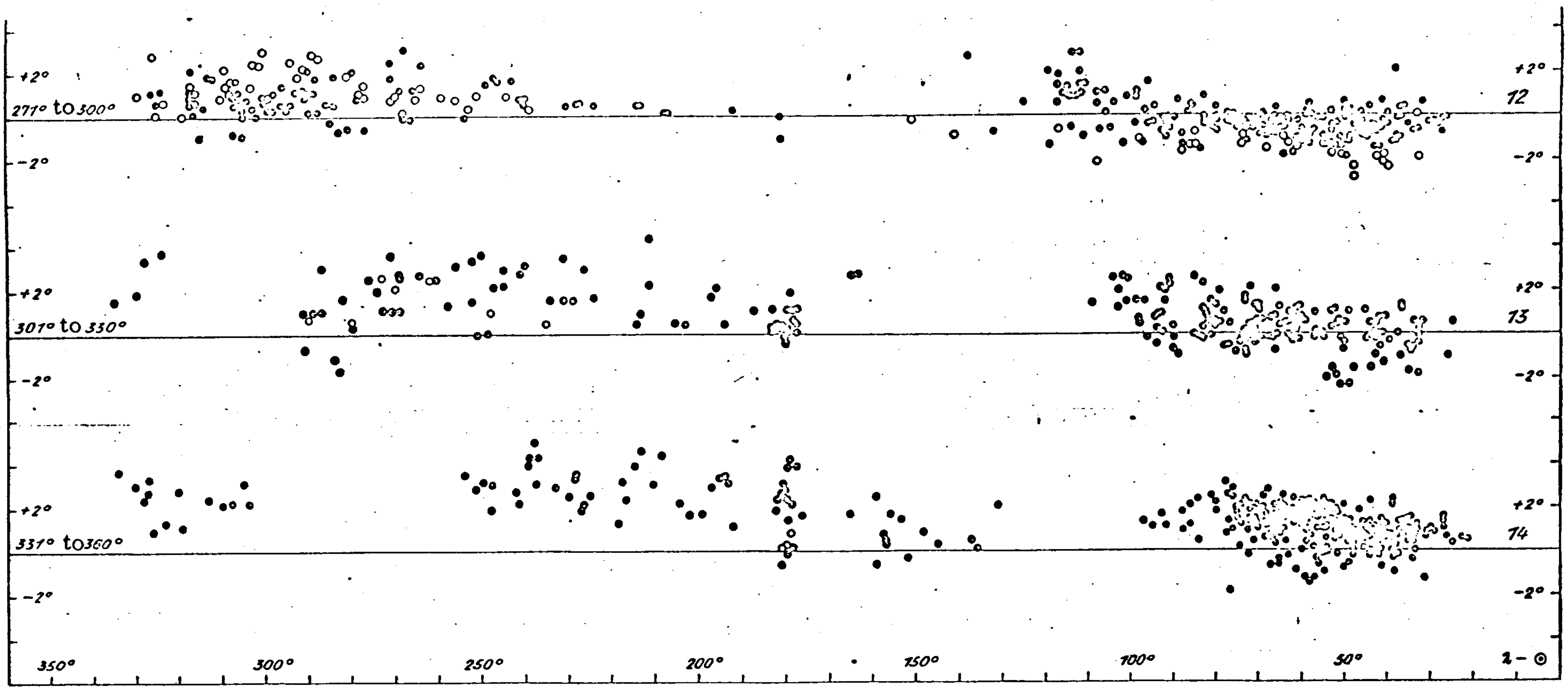


TABLE 2. C. HOFFMEISTER: THE AXIS OF THE ZODIACAL LIGHT AND THE CONSTANTS OF ITS PLANE OF SYMMETRY



Axis Points Observed for the Zodiacal Light in Fourteen Series Taken Throughout the Year. Abscissa: Elongation to the East of the Sun; Ordinate: Ecliptical Latitude; at the Left: Limits of the Solar Longitudes for Each Group. ● = Observed from the Northern Hemisphere. o = Observed from the Southern Hemisphere.



(d) Disturbances of the Light Axis by the Milky Way.

As we can see from Figure 5, the annual sinusoidal lines for the same easterly and westerly elongations in many cases show very different amplitudes i'. We have attributed this phenomenon to a disturbance caused by the Milky Way. In Figure 8, this effect has been shown schematically for a sinusoidal line with $\Omega' = 90°$. The galactic brightness increases with decreasing galactic latitude in the direction of the arrows, and the light axis of the ZL is displaced in the same direction so that the amplitude of the oscillation is reduced in this case. This will generally be the case when $0° < \Omega < 180°$. On the other hand, if the nodes lie between 180 and 360°, the amplitude is increased. In the vicinity of $\Omega' = 0°$ and 180°, the total effect will be zero and only a deformation of the sinusoidal line will occur.

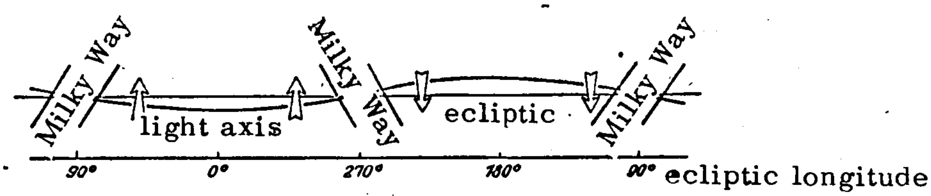


Fig. 8. Disturbance of the Light Axis by the Milky Way.

In reality, the Ω' for negative elongations lie between 60° and 160°, so that the amplitudes must therefore appear sharply reduced (nearly 350° and 0° for positive elongations), and the disturbance nearly disappears. To average the two partial values of i', for lack of a better method we have assigned weights $p_1 = |\cos \Omega'|$. Hence, in the regular case the practically undisturbed value belonging to the positive e has the greater weight. There is hardly any other method possible, since we do not know how the disturbance works in detail and especially whether it is produced by the general galactic sky brightness or only by the boundary regions of the actual Milky Way which are visible as such. Therefore, we cannot even assume that Ω' itself remains undisturbed; we can assume with a certain degree of probability however, that in this respect the affects compensate one another.

It is surprising to find that the observations of Jones show the amplitude effect to be much more pronounced but with reversed sign. It is highly likely that this can be attributed to the fact that Jones did not observe axis points but only boundary lines. It is a known fact that the superposition of a veil of light does not extend the boundaries of the ZL but rather reduces them. As a rule, we see the ZL as being very broad; with weak moonlight, on the other hand, it takes on the narrow shape preferred by many observers (cf. [1, Figs. 12 and 13]). If the veil is unilateral or nonuniform, the outlines on the brighter side of the veil are restricted more sharply than on the other side, i.e., they are displaced toward the axis. The Milky Way therefore has a repulsive effect on the outlines and an attractive effect on the axis points.

It should also be mentioned that in Jones' data as well as in the newer series, the disturbance is nearly insignificant at the minimum elongations, reaches a maximum at average elongations, and again decreases sharply at e = 100°. The former observations can easily be explained by the fact that at small solar distances the brightness of the ZL is much greater than that of the Milky Way. It is more difficult to interpret the decrease of interference at greater elongations. If this might somehow be related to the fact that the observer himself avoids a confusion with the weaker boundary portions of the Milky Way at those points where the ZL is very weak, it would be possible to see this as an indication that the effect is produced predominantly by these boundary portions and less by the gradient of brightness at higher galactic latitudes. /66

In addition, the findings for the Gegenschein (especially the differences between our observations and the Harvard observations) can be interpreted as the effect of galactic brightness, since we ourselves always observed the center of the light while the Harvard observers usually drew the outlines. It may be that the effect is not insignificant in this case as well, but so long as the opposite has not been proven, the explanation given above may be considered even more probable. It should be noted that the center of the light in the Gegenschein is better defined than the axis points of the ZL at other elongations, and that apparently the effect as such has less of an effect in the weaker parts of the ZL, and finally that the observer looking at the Gegenschein, whose position must be given in two coordinates, must be particularly careful to avoid disturbances caused by the Milky Way, i.e., to make his observations only when the Gegenschein is located completely outside of the detectable boundaries of the Milky Way. The significance of a positive effect of this kind can, in our opinion, be found in the difference of about 0.5° between the Harvard values and the ones that we found for i' for the elongations 160° and 135°. This figure is twice the effect which would be expected for the Gegenschein at about 0.25°. The average values used to determine the parallax and the distance could be practically free of disturbance.

7. Summary of the Results and General Remarks

A significant portion of the present investigations was devoted to the errors underlying the observations of the ZL axis owing to disturbing influences. The sources of these disturbances are the following:

1. Atmospheric extinction
2. Twilight
3. Atmospheric skylight

4. Milky Way and Galactic Haze.

The errors produced by 1 to 3 can be avoided if we limit ourselves to steep positions of the light axis. The earlier limit for maximum inclination of 15° to the vertical was found on the basis of the new experiments to be satisfactory. Observations made under less satisfactory geographical conditions serve to determine the degree of error. Separation of the errors produced by atmospheric effects on the basis of their different causes is generally impossible, and only the total effect is of significance for the problem under discussion here. Analysis led to the drawing up of a table of errors, with which the observations can be corrected as long as extreme conditions are avoided. Within the range covered up to 35° axis inclination and with omission of the portions of the ZL located in the immediate vicinity of the horizon, the errors in ecliptic latitude reached a maximum value of 1°. In a special case, that of the Gegenschein, it was possible to show that at average zenith distances extinction plays the predominant role while at larger zenith distances it is the skylight which does this. Compensation occurs at approximately 60° ZD.

Disturbance caused by the Milky Way as well as by groups of stars and individual bright stars cannot be avoided in the manner just described. One can be careful to exclude obviously disturbed regions of the light axis, but it turns out that this deals with only a portion of the disturbance. In general, the latter can be described so that where the ecliptic forms a sharp angle with the galactic equator, the light axis is drawn inside the angle. However, it appears that not only the edges of the Milky Way, but also the grad ients of galactic brightness at higher latitudes have an effect. In any case, the disturbance can be detected at only a few points. Its effect on the final result was negated to a considerable extent by suitable selection of the weights of the individual observation groups. It is remarkable that the effect of the Milky Way is reversed (i.e., appears to have a repulsive effect) if the outlines are observed rather than the axis points.

The latitudinal effect assumed by Schoenberg and Pich was not confirmed. An apparent effect of this type has been interpreted as the effect of a selection principle.

These preliminary observations can now be used as the basis for a nearly complete axis determination in a new determination of the constants of the plane of symmetry of the ZL. The relationships of the plane of symmetry to the planes of the orbits of the major planets, found in 1932, was completely confirmed. A slight deviation was found only at the smallest radii, while no significant effect was found for Mercury. The relationship to the orbits of Venus, Earth and Mars, on the other hand, is very close as far as the nodes and inclination are concerned, and the assumption of a close relationship with the orbit of Jupiter made /67 it possible to present the observations of the outer zodiacal ring without any contradictions. However, these observations have made a relatively pronounced eccentricity of the outer ring possible. Another new finding is the fact that the less light-effective parts of the zodiacal light located between the orbits of the Earth and Mars can obviously be assigned to the outer portion. Because the observations showed that the optical density in this region can at least no longer decrease with increasing radius vector, we see that the mass density must increase. This means that the outer portion gains in extent considerably with respect to the previous view and thereby loses the uniformity which it formerly had by virtue of its relationship to the orbit of Jupiter. This means that the properties of its plane of symmetry must resemble those of the inner portion, with the difference that the portions which are not associated with the plane of Jupiter's orbit are less light-effective, so that the phenomena as a whole are simpler than in the inner portion.

Physical questions have not been discussed. As was mentioned earlier [1, pp. 87-91], the value Δm shown in Figure 6 (obtained here by purely astrometric means) is determined from the phase law and can be used to test the phase law stated in the theory in a manner described in greater detail in [1]. Before undertaking a new study in this direction, we must await publication of further photometric observations of the ZL in order for this experiment to proceed jointly with another devoted to a redetermination of the density function.

The author has gained the impression that the present treatment has nearly reached the limit that can be set at the present time with regard to determination of the position of the zodiacal light. A further refinement of the data is impeded primarily by the difficulties arising out of the disturbances of the light axis. Of these disturbances, those caused by the Milky Way are by far the most important because (in contrast to the atmospheric disturbances) they cannot be excluded by any special observation procedure.

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